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Model Correlation and Damage Location for  
Large Space Truss Structures:  
Secant Method Development and Evaluation

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Final Report

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## Abstract

On-orbit testing of a large space structure, like Space Station Freedom, will be required to complete the certification of any mathematical model for the structure dynamic response. Prior to assembly in orbit, model verification will occur only at the substructure level or with scale model technology. Ground tests of the complete structure are not possible.

The process of establishing a mathematical model that matches measured structure response is referred to as "model correlation." Most model correlation approaches have, as their essence, an identification technique to determine structural characteristics from measurements of the structure response.

The current research program, under Grant NAG-1-960, approached this problem with one particular class of identification techniques - matrix adjustment methods - which use measured data to produce an optimal update of the structure property matrix, often the stiffness matrix. Previous research had led to new methods for identification to handle problems of the size and complexity expected for large space structures. Further development and refinement of these "secant-method" identification algorithms was undertaken. Also, evaluation of these techniques in an approach for model correlation and damage location was initiated.

Efforts for development and evaluation of secant methods for structure identification have produced understanding in a broad area now termed optimal-update identification or optimal matrix adjustment. This is the major result of this grant effort. Constrained optimization formulations for updating the property matrices of a model of a structure using measured dynamic response data have been developed and examined. In addition, these techniques are applicable to other problems in which multiple constraints can be specified for a matrix updating problem, for example, control gain calculations.

Evaluation of these techniques for the problem of damage detection is still in progress, however. Improved understanding resulted from the development efforts and evaluation studies under this grant but additional efforts were defined as a result. In particular, attention to the problem of sensor placement, among others, is needed to produce the best mode shape information for the identification process.

As a result of this research effort, ten publications have, or will, appear. In the summary section, these publications are listed. Abstracts are reproduced where they are available.

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## 1.0 Introduction

On-orbit testing of a large space structure, like Space Station Freedom, will be required to complete the certification of any mathematical model for the structure dynamic response. Prior to assembly in orbit, model verification will occur only at the substructure level or with scale model technology. Ground tests of the complete structure are not possible.

The process of establishing a mathematical model that matches measured structure response is referred to as "model correlation." Most model correlation approaches have, as their essence, an identification technique to determine structural characteristics from measurements of the structure response.

Once the mathematical model is updated and accepted as the correlated model of the undamaged structure, monitoring the structure for damage is equivalent to the model correlation problem just completed. Again, a mathematical model of the structure is developed using response measurements. This current model is compared to the correlated model for the undamaged structure to locate areas of disagreement, which indicate that damage has occurred and its location.

A research program, under Grant NAG-1-960, approached these problems with one particular class of identification techniques - matrix adjustment methods - which use measured data to produce an optimal update of the structure property matrix, often the stiffness matrix. Previous research [R-177] had developed new methods for identification to handle problems of the size and complexity expected for large space structures. Further development and refinement of these "secant-method" identification algorithms was undertaken as a part of the research program. Also, evaluation of these techniques in an approach for model correlation and damage location was initiated.

This report presents the research program results for a period from February 1, 1989 to January 31, 1991. Results that have appeared, or will appear, as a part of the technical literature in the form of journal papers or conference papers [R-107 to R-110, R-112, R-190 to R-193] are referenced in the following, but not reproduced in this text. Primarily, the report includes an extensive review and bibliography. Summaries of the findings which have been published in other forms are included as well.

The following introduction sections provide a brief discussion of recent developments in space structure identification and damage location. Chapter 2 is an extensive review of identification techniques and damage location in the aerospace literature, as well as in the literature of other engineering disciplines. Chapter 3 presents a summary of optimal matrix adjustment methods. New developments, including new applications for optimal-update identification, are discussed in Chapter 4. Chapter 5 summarizes results of the evaluation studies for performance of the methods for model correlation and damage location. These evaluations were conducted with simple spring-mass models and using a laboratory truss structure. Chapter 6 is the summary and conclusions.

### 1.1 Space Structure Identification

Successful model correlation or damage location depends on the ability to identify structural characteristics. A review of on-orbit identification of large space structures was prepared by a task committee whose stated goal was to "develop a state-of-the-art report on methods for identification of large structures in space" [R-178]. Their recommendations included a call for experimental evaluations and comparison studies of identification methods.

More recently, a workshop [R-179] was held to discuss current programs in on-orbit identification. Presentations ranged from reviews of identification techniques to reports of experimental results to discussions of nonlinearities and model uncertainties. Some talks on damage assessment are included, as well.

### 1.2 Damage Location

This year, a second USAF/NASA Workshop [R-180], addressed on-orbit identification for space structures. A focus on experimental programs and on "health monitoring" was adopted. From this workshop an assessment of the state-of-the-art is possible.

In identification, two ideas were prominent. First, uncertainties in the structure model may be addressed by establishing the set, or sets, of system models that best represent the structure response. Optimal controllers are designed to be robust with respect to the entire set of models for the structure. Second, learning system identification may use all previous experimental results in an intelligent way to complete the model identification. Additional experiments can be tailored to supplement the previous tests.

Several ground experiment programs compared the performance of various modal identification techniques. These methods use measured structure response to produce modal parameters; frequencies, damping factors, mode shapes and modal participation factors. The Eigensystem Realization Algorithm (ERA) [R-181, R-77, R-76, R-78] was cited as the technique with the strongest performance in several programs. Refinements of and improvements to ERA were presented, also.

Primarily, two types of structures were examined in the various research programs - truss structures, either deployable or erectable, and reflector structures. Characteristics of the response are different for the two types of structures, but overall they present many of the same challenges: closely-spaced, low-frequency modes, low damping, possible nonlinearities, and symmetry, among others.

Various sensing devices are under investigation to explore their capabilities for identification including optical sensing, laser sensing and imbedded fiber-optic sensing. Active members of truss structures may also be used for identification.

"Health monitoring" encompasses sensor failure and other system failures along with structural degradation and failure. Results from the experimental program detailed in Chapter 5 of this report were the only experimental results presented [R-109] which focussed on damage location.

On-orbit experiments which are underway or which are planned were highlights of the workshop:

1. Recent parabolic flight profiles of a KC-135 aircraft were used to achieve free-fall conditions and free-free tests of two truss structures. With results of the flight tests compared to results of ground tests designed to simulate free-free conditions, an evaluation of ground test techniques is anticipated. This program is a continuing effort of the Structures Division of the Flight Dynamics Laboratory at Wright-Patterson Air Force Base. The program is focussed on system identification and active control.
2. Flight dynamics experiments for modal identification of the Low Altitude Compensation Experiment (LACE) satellite (which is currently on-orbit) are being conducted by the Naval Research Laboratory. Three flexible, deployable truss booms are the primary structure of the satellite. Using ground-based laser measurements and on-board sensors, identification of all six of the first flexible modes is feasible. The actual experiments will occur later this year.
3. The Space Station Freedom Structural Characterization Experiment is a flight experiment under development by NASA Langley Research Center. Techniques for on-orbit modal testing can be applied to various build-up configurations of the Space Station for identification of the properties of this large flexible space structure. Analytical studies are concurrently underway to support the development effort and to evaluate techniques for on-orbit identification.
4. The MIT Space Engineering Research Center is designing the MODE (Middeck Zero-Gravity Experiment) experiment to identify linear and nonlinear dynamic characteristics of contained fluids and of very flexible structures. The experiment is designed for the middeck of the space shuttle. Model coupling, as well as nonlinear model techniques, may be evaluated as a result of this experiment in which ground test results will be compared to flight experimental data.



The field of structure identification still contains many unaddressed or incompletely addressed issues. Foremost among these is the issue of uncertainty in the identified model. How to reduce the uncertainty, how to quantify the uncertainty, how to use identified models even with uncertainties are all questions to be explored. Using system identification techniques for damage location or "health monitoring" is another major issue. Of these two, the second is explored in more detail in this final report, but the first is considered as well.

## 2.0 Background

In the subsequent sections, background material is presented on structure identification and damage location. First, optimal matrix updates for structure property matrices will be reviewed. These techniques led to the secant methods presented herein. General background on other techniques for structure identification and on techniques for modal identification follows. Then reviews of test/analysis correlation and damage location research close the chapter. Damage assessment (or location) research, which employs structural parameter identification, is presented, along with damage location research which uses dynamic testing, but not identification.

### 2.1 Optimal Matrix Updates

#### 2.1.1 Early Work

Methods based on matrix approximation ideas have been in use in structural dynamics since the late-1960's. Rodden [R-132] used results from ground vibration tests to derive the matrix of structural influence coefficients. The resulting underdetermined system of equations was solved using the generalized inverse, then further adjustments helped to satisfy physical constraint conditions.

Brock [R-133] examined the problem of determining an optimal matrix to satisfy a set of measured conditions and which may also possess certain matrix properties. The symmetric matrix is found which most closely satisfies conditions with the measured data. In the current research, these ideas are extended. In addition to symmetry, matrix properties such as positive definiteness may be imposed. Initial information about the matrix (for example, an initial model) can be incorporated by finding the symmetric property matrix as above, subject to having it as close as possible to the initial model as well.

The problem of incorporating initial model information can be framed in a different way, with the updated matrix as close as possible to the initial matrix and also satisfying conditions with the measured data and symmetry. This alternate formulation imposes a more rigid constraint with the measured data (which is likely to contain errors). Further discussion of this point follows in Chapter 3 which includes more detail on these two views of optimal-update techniques.

For the structural identification problem with measured frequencies and mode shapes, and assuming an initial model of the mass matrix is known to be an accurate representation of the mass distribution, the stiffness matrix must satisfy conditions with the measured modes. An initial model of the stiffness might be from the finite-element model, with the initial mass matrix as the finite-element mass matrix.

#### 2.1.2 Recent Optimal Updates

The latter problem formulation of above is the basis of a sequence work in optimal-update identification which started with the work of Baruch and Bar Itzhack [R-54]. In their work on orthogonalization of measured modes, Baruch and Bar Itzhack developed a minimal matrix adjustment for the structure stiffness matrix, using measured frequencies and mode shapes. Berman and Nagy [R-65] adopted a similar approach in an effort to improve analytical models using test data, addressing the possibility of both mass and stiffness matrix updates.

Baruch [R-56 to R-62] and Berman [R-55,R-64,R-66 to R-74] followed their initial works with an exchange of ideas and extensions of the techniques. Wei [R-137,R-139,R-141] provided further clarifications and extensions, most recently combining the mass update and stiffness update in a single constrained minimization formulation [R-141].

With each of the preceding methods, the sparsity (the zero/nonzero pattern) of the original stiffness matrix is not preserved in the adjusted stiffness matrix. If the original model corresponds to a reduced model with few zero elements, this might not be a difficulty. However, for the similar adjustment of an original finite-element stiffness matrix having considerable sparsity, the nonzero elements created in the adjusted stiffness matrix correspond to physical load paths that do not exist in the actual structure. This reduces confidence in the adjusted model.

Kabe [R-85,R-86] addressed these concerns and presented a stiffness matrix adjustment procedure which preserves the physical connectivity of the original model in the updated stiffness matrix. An updated stiffness matrix that is "closest" to the original matrix, yet satisfies constraints imposed by the measured modal data, symmetry and the sparsity of the original matrix, is found. The measure of "closeness" is a sum of the change percentages for each nonzero element of the stiffness matrix rather than the matrix Frobenius norm used in previous techniques.

Kammer [R-90] presented a stiffness matrix identification method with the same objectives as in Kabe's work. Producing equivalent results with similar computational effort, Kammer's method is an important reformulation of Kabe's method that permits more flexibility in defining weights in the objective function.

Prior to the current research effort, a new technique for optimal stiffness matrix identification was initiated by the authors [R-177]. A multiple-secant generalization of the Marwil-Toint [R-187, R-188] update of computational linear algebra produced an identification technique which preserves the sparsity of the original model, without excessive storage requirements or computational effort. Details of the development of this new method and other related optimal-update techniques are presented in Chapter 3.

## 2.2 Mode Shape Expansion and Orthogonalization

Throughout the previous discussions it has been assumed for each of the observed modes, that all modeled degrees of freedom were accessible to measurement. Due to instrumentation costs, limited data handling capabilities and inaccessibility, the actual number of degrees of freedom is far less in practice - rarely exceeding 10% of the total number that may be modeled.

Guyan [R-183] presented the premiere work on model reduction to accommodate this situation. The mathematical model of the structure dynamic response is a reordered, partitioned eigenvalue problem, corresponding to the measured and unmeasured degrees of freedom.

Rearranging the second of two resulting matrix equations produces an expression for the unmeasured degrees of freedom of the mode shape which can be used to reduce the mathematical

model representation to one with only the measured degrees-of-freedom. In other words, all unmeasured degrees-of-freedom are eliminated from the model by expressing them in terms of the measured degrees-of-freedom using static equilibrium equations.

Guyan reduction may lead to inaccurate models in cases where the neglected dynamic terms are significant, i.e., significant mass or relatively high frequencies. Conti [R-188] presented a higher order generalization of the Guyan method to address these shortcomings. He expanded the representation of the unmeasured degrees-of-freedom with a converging power series to allow efficient computations and more accurate reduced models.

An alternate reduction technique was presented for "test-analysis-model (TAM)" development by Kammer [R-89,R-91] and separately for "system equivalent reduction and expansion (SEREP)" by O'Callahan, Avitabile and Riemer [R-100,R-102 to R-105]. A vector of physical displacements, i.e., a mode shape vector in the physical coordinates, is expressed as a linear combination of analytical model mode shapes.

After reordering and partitioning as before, the unmeasured degrees-of-freedom can be represented in terms of the measured degrees-of-freedom by solving the first of the partitioned equations for the modal coordinates using a generalized inverse.

For model reduction, once the unmeasured degrees-of-freedom are expressed in terms of the measured values, substitution into the original eigenproblem produces a new model with "reduced" stiffness and mass matrices. The new stiffness and mass matrices are dimension  $r \times r$ , with  $r$  as the number of measured degrees-of-freedom.

It is often useful to extrapolate values for the unmeasured degrees of freedom based on both modeled dynamic information and the available measured degrees-of-freedom, without producing a reduced mathematical model. This extrapolation can be accomplished by reversing the reduction process, thus, "expanding" one mode at a time. The set of expanded mode shapes is not likely to be orthogonal with respect to the analytical model mass matrix, so a subsequent orthogonalization procedure is necessary to introduce this property to the set of expanded modes. Several

optimal-update identification techniques assume orthogonality of the measured mode data with respect to the analytical mass matrix.

Baruch and Bar Itzhack [R-54] presented an optimal orthogonalization technique which adjusts the measured set of modes to the closest set which is orthogonal to the structure mass matrix. This orthogonalization problem can also be framed as an optimal matrix adjustment technique. The weighted optimization technique performs symmetric adjustment of the mode shape vectors.

Prior to the work of Baruch and Bar Itzhack, Targoff [R-127,R-129] and Rodden [R-128] discussed orthogonality of measured modes and methods to "correct" the measured modes. Gravitz [R-130] also presented an approach for orthogonalization of measured modes.

Chu, et. al. [R-114] discussed the pitfalls of using a mass orthogonality check for mode shapes derived from measured data. Many of the previously cited techniques were tested to determine the consequences of their application in circumstances leading to structure identification.

Finally, Kabe [R-87] proposed a procedure "to improve the mass weighted orthogonality of measured mode shapes." Different excitation levels and locations can be used to improve the isolation of the modes and therefore improve the result.

## 2.3 Structural Identification

A discussion of techniques for structural identification which do not employ matrix optimization is appropriate at this point. Ultimately a hybrid of these alternate techniques and optimal updates may be the most effective.

### 2.3.1 Sensitivity Methods

Adelman and Haftka [R-165] recently presented a review of research in the area of sensitivity analysis for structures. A variety of applications for sensitivity derivatives has encouraged work in this area. Optimal control and structural optimization use derivatives to establish effective

search directions for optimal solutions. Efficient computation of derivatives is also studied to improve computationally intensive optimization procedures. Recently, analytical model improvement using sensitivity derivatives has received attention, along with applications in design methodology.

For structural identification using sensitivity derivatives, a set of physical parameters (densities, moduli of elasticity, areas, lengths, masses) is chosen to represent the structure. Elements of the dynamic model property matrices, mass, stiffness or damping, are functions (often nonlinear) of the physical parameters. In turn, eigenvalues and eigenvectors of the response can be expressed as functions of the parameter set. Using local linearization, the functional relationship of system properties to the parameter set is determined.

Adjustments in the physical parameter set are identified using the dynamic response of the structure and the sensitivity derivatives. The update is accomplished iteratively or optimally, depending on the number of physical parameters and on the number of measured properties, which set whether the system of equations is overdetermined or underdetermined.

Creamer and Hendricks [R-80] presented a technique to identify physical parameters of a structure using measured natural frequencies and sensitivity derivatives. They examined the difficulties encountered with symmetry in the structure.

Calculation of eigenvalue and eigenvector derivatives has received considerable attention. Lim, Junkins and Wang [R-79] recently re-examined the problem of eigenvector derivatives presenting an improved method for non-self-adjoint systems.

Property matrices and modal quantities are not the only structure models which can be examined for sensitivity with respect to the physical parameter set. Chon [R-166] discussed strain energy sensitivity to joint stiffness.

Applications of structural identification using sensitivity derivatives are available in the literature. Brillhart, et. al. [R-174,R-175] reported results of modal tests and test/analysis correlation for a transfer orbit stage structure. Additional applications are cited in the sections which follow.

### 2.3.2 Eigenstructure Assignment

Another class of structure identification techniques grew out of the controls research on eigenstructure assignment. Srinathkumar [R-163] presented a method for optimal control design which determined the appropriate control (an adjustment to the control/plant system) to result with a particular system response (or eigenstructure). Andry, Shapiro and Chung [R-164] extended these results.

For structure identification, eigenstructure assignment theory is applied to determine the fictitious control which would be required to produce the measured dynamic response with the initial model system. This fictitious control is translated into adjustments to the initial model appropriate for the observed structure response. Minas and Inman [R-75] and Zimmerman and Widengren [R-39] presented variations of this idea.

### 2.3.3 Other Methods

While this review is extensive, it is not comprehensive due to the numbers of structural identification techniques which have been presented over the last 20 years. Several ideas deserve mention, though. Some of them are covered in this section.

Rajaram, et. al. [R-83,R-84] presented an identification technique which is notable in that all elements of the stiffness, damping and mass matrices of a linear model can be determined with the size of the model as the only initial information. Measurements of displacement, velocity and acceleration are required at each degree-of-freedom to provide enough data to produce an over-determined system of equations to solve for the elements. Symmetry is not presumed. This technique is not practical with current measurement technology, but demonstrates one extreme in identification.

Chen and Garba [R-121] used a perturbation analysis to formulate a structure identification procedure. White and Maytum [R-146] used scaled submatrices, finite elements or groups of elements, to adjust the response model. Creamer [R-81] used techniques for modal identification with the frequency response function and extended them to determine structure property matrices.



Nonlinear systems have been considered by Mook [R-98], although the techniques are developed only for simple single degree-of-freedom systems at this time. Multi degree-of-freedom nonlinear systems were considered by Gifford and Tomlinson [R-162].

Kuo and Wada [R-154,R-156] have studied testing flexible structures with multiple, different boundary conditions to enable ground testing and to enhance identification. They also have employed nonlinear sensitivities for identification [R-161].

Modal identification, determination of modal parameters - frequencies, mode shapes, damping and modal amplitudes, is a related area of research which is, in turn, related to modal testing. Juang and Pappa [R-184] prepared an extensive review of modal testing and identification with a focus of control of structures.

Additional references on modal identification and modal testing have been omitted from this summary. Identification with statistical methods and identification of continuous structures without using a spatial discretization are also excluded. While these topics are related to structure identification with optimal matrix updates, they are not essential to the subsequent discussions.

#### 2.4 Test/Analysis Correlation with Structure Identification

Many applications of structure identification for test/analysis correlation (or model verification) are presented in the literature. Optimal-update identification is employed in some cases; sensitivity techniques are used in others.

Matzen [R-23,R-24] has studied structural identification for shear buildings and indeterminate truss structures. His approach for system identification uses sensitivity techniques to solve for the elements of mass, damping and stiffness matrices of a discretized model of the structure rather than for physical parameters of substructures (i.e., elastic modulus, cross-section area, etc.). While damage location was not addressed specifically in this work, structural identification can play a key role in damage detection.

Detection of changes in structural parameters using system identification is the topic investigated by Agbabian and Masri, with others [R-20,R-41]. A system identification technique, simi-

lar to that used by Matzen, is evaluated in the presence of noisy measurements. Simulated data for an idealized bridge structure is used in the most recent studies.

Douglas and Reid [R-182] also investigated system identification of a bridge structure. A five-span four-hundred-foot long, reinforced concrete box girder bridge was tested extensively. System identification techniques were employed to determine stiffness properties, among others, of the structure. The authors concluded, "Thus, carefully conducted dynamic tests of full-scale highway bridges provide detailed information about the structure (and) ... shed much experimental light on the soil-structure interaction aspects of the problem..."

Optimal-update identification was used for test/analysis correlation of a welded plate structure by Lapierre and Ostiguy [R-115]. Hashemi-Kia [R-38] used optimal-matrix updates and modal tests to investigate characteristics of an elastic wing model.

Caesar [R-116,R-117] attempts to unify the works of Berman and Baruch in a comprehensive correlation approach. Simulated studies with a 27-dof structure are presented. Link [R-142] also investigates optimal-update methods for structural model matrices for test/analysis correlation.

## 2.5 Damage Location

The subjects of damage detection and damage location have received considerable attention in recent years. Much work has been done in the Civil Engineering community, although several researchers in the aerospace community are active as well.

### 2.5.1 Damage Location with Structure Identification

In a series of work, Yao and coauthors [R-10,R-12,R-13,R-15,R-16] have explored the problem of damage detection in buildings. Vibration testing and identification techniques were used to examine changes in the damping of the structure. A conflict between damage detection and structural verification was discussed. A damaged or flawed structure may still satisfy system requirements and, thus, may be verified as acceptable. The complexity of the damage location problem prompted Yao to suggest artificial intelligence as an approach for damage assessment.

An excellent series of work focussed on locating damage in fixed offshore platforms using ambient excitation and system identification. Rubin and Coppolino [R-28,R-29,R-17] reported success with deducing which members of the platform were damaged for key structural elements. "The laboratory and field testing to date strongly suggest that the method (of ambient vibration testing) can be developed to become viable for cost-effective contribution to an overall structural inspection program."

Hajela and Soeiro [R-48,R-49,R-50] have recently studied damage assessment using optimal-matrix identification. Their work is focussed on several types of structures, but primarily on truss-type structures and on composite materials. Simulation studies were conducted to evaluate proposed approaches.

Stubbs and coauthors [R-37,R-152] have also considered using system identification for locating damage. Simulation studies on simple models have been used to demonstrate the techniques.

#### 2.5.2 Damage Location with Dynamic Testing

Kircher [R-176] studied vibration testing of buildings. Similar structures were tested to determine if similarities existed in the frequency response. Similar substructures (in this case, different floors of the buildings) were tested as well. The dynamic response of buildings of "identical" design was comparable, with the most similar response occurring for buildings built by the same contractor during the same construction period. Kircher also reports that "Virtually identical power spectral density functions are obtained from acceleration measurements taken at the same location in the same building at different times." Therefore, changes in the response could indicate changes in the structure, perhaps due to damaging earthquakes.

Tracy and coauthors [R-34,R-35] consider damage in composite materials. First, experiments were used to examine the changing dynamic response in delaminated structures. Modal testing was then investigated as a tool to determine the damage by using the dynamic response.

Complex aerospace structures can be tested to evaluate structural integrity, as well. West [R-30,R-33,R-46,R-47] and Pacia [R-43,R-45] have been involved with experiments with damaged aircraft structures, including a small plane, an F-16 wing, and the shuttle orbiter. Modal testing was used to detect damage in each of these studies by examining changing frequency response.

### 3.0 Optimal-Update Identification

In this section we present developments in secant-method identification. Further details of the developments can be found in References R-177, R-108, R-110 and R-190. This grant supported the second, third and fourth of these publications.

#### 3.1 Optimal-Update Identification

Optimal-update identification is an approach to produce, through the solution of a constrained optimization problem, adjusted or updated property matrices (i.e., mass, damping or stiffness matrices) that more closely match the structure modal response. Along with matching the measured modal data, qualities of symmetry, closeness to the initial model, positive definiteness, and preservation of structure connectivity are imposed in various techniques.

In optimal-update identification one of two views may be adopted to establish the constrained minimization problem which produces the adjusted property matrix. The first view was used by Baruch and Bar Itzhack [R-54] and by Kabe [R-85] in their methods for stiffness matrix adjustment. Generally, this view can be formulated as

$$\begin{aligned} \min \|A - A_0\|_F \\ \text{subject to } AS=Y \text{ and } A = A^t, \end{aligned} \quad (3.1)$$

where

A is the nxn adjusted property matrix,  
A<sub>0</sub> is the nxn initial-model property matrix,  
S, Y are nxp matrices that define constraints with the measured data,  
n is the number of degrees-of-freedom of the model,  
p is the number of matrix constraint equations,  
t indicates the matrix transpose, and  
 $\|E\|_F = (\sum_{i,j=1}^n |e_{ij}|^2)^{1/2}$  is the Frobenius norm.

Here a matrix closest to the original model matrix is found. It is, however, the closest matrix that is symmetric and that also satisfies the constraints of the measured data. Additional constraints

may be imposed, including sparsity (the zero/nonzero pattern) and positive definiteness. Weighting matrices may be added in the objective function of the minimization problem as well. Specific techniques that stem from this first viewpoint vary in the measure of closeness used and in which constraints are included.

In general, these algorithms exhibit several characteristics that make them attractive for structure identification. Minimal data is required in terms of the number of modes [R-85] and applications to model refinement for actual structures have been accomplished [R-66, R-115]. However, experience indicates that constraints imposed with measured data which are inconsistent with the other constraints of the technique can lead to poor results. In the case of data inconsistent with the desired sparsity pattern, an updated model which is not positive definite may occur [R-85]. One option is to introduce the measured-data constraints as a "soft" constraint in the objective function, thus finding the closest matrix which most nearly matches the dynamic response.

The second view for framing the optimal-update problem follows a different philosophy. This view was adopted by Brock [R-133] in his development of optimal matrices describing linear systems and by Kammer [R-90] in his projector matrix method. The second view starts with symmetric minimizers for the measured-data constraint and imposes further constraints from there. Generally, this view is formulated as

$$\begin{aligned} \min \|AS - Y\|_F \\ \text{subject to } A = A^t \text{ and other constraints.} \end{aligned} \quad (3.2)$$

Although optimal-update identification algorithms can be developed for mass, damping and stiffness property matrices, stiffness matrix identification has been most widely explored. In the next section we present a series of stiffness matrix adjustment techniques that have all been developed with an optimal-update identification approach.

### 3.2 Stiffness Matrix Adjustment

A review and summary of stiffness matrix identification techniques under the general approach of optimal updates serves to illustrate the two viewpoints described previously and allows a discussion of performance characteristics. General observations about these algorithms are included. Further discussion and proofs are presented in the references supported by this grant.

For this approach, consider at first an  $n$  degree-of-freedom structural model of the free, undamped vibration,

$$[M_a]\ddot{x} + [K_a]x = 0 \quad (3.3)$$

$M_a$  is the  $n \times n$  symmetric, positive definite, original-model mass matrix,  
 $K_a$  is the  $n \times n$  symmetric, original-model stiffness matrix,  
 $x$  is the  $n \times 1$  vector of displacements, and  
 $\ddot{x}$  is the  $n \times 1$  vector of accelerations.

Baruch and Bar Itzhack [R-54], in their work on orthogonalization of measured modes, presented a technique for stiffness matrix adjustment which exemplifies the first viewpoint. The stiffness matrix for a structure is improved using measured frequencies and mode shapes. A similar technique was adopted by Berman and Nagy [R-66] in their efforts to improve analytical models, but they considered both mass matrix updates and stiffness matrix updates. The stiffness matrix adjustment problem is formulated as

$$\begin{aligned} \min \|M_a^{-1/2}(K - K_a)M_a^{-1/2}\|_F \\ \text{subject to } KS = M_a S \Omega^2 \text{ and } K = K^t, \end{aligned} \quad (3.4)$$

where

$K$  is the  $n \times n$  adjusted stiffness matrix,  
 $\Omega^2$  is the  $p \times p$  matrix of measured eigenvalues,  
 $S$  is the  $n \times p$  matrix of  $p$  corresponding measured mode shapes, and  
 $\|E\|_F = (\sum_{i,j=1}^n |e_{ij}|^2)^{1/2}$  is the Frobenius norm.

A closed-form solution involving only matrix multiplications exists and was given by Baruch and Bar Itzhack [R-54]:

$$K = K_a + M_a S \Omega^2 S^t M_a - K_a S S^t M_a - M_a S S^t K_a + M_a S S^t K_a S S^t M_a. \quad (3.5)$$

Positive definiteness is guaranteed in the adjusted stiffness matrix if the original stiffness matrix is positive definite. However, the sparsity pattern of the original model is lost in the update. For dense models which arise from reduced system models, this may not be an issue.

Sparsity is preserved in the adjusted property matrix by adding an additional constraint to the problem. Kabe [R-85] presented a technique for stiffness matrix adjustment which, when expressed to compare with the formulations of this work, is stated as follows:

$$\min \sum_{i=1}^n \sum_{j \in \text{adj}(i)} \frac{(K_{ij} - K_{ij}^*)^2}{(K_{ij}^*)^2}$$

subject to  $KS = M_s \Omega^2$  and  $K = K^t$ ,

(3.6)

The research sponsored by this grant included further development of a sparsity preserving update which is an analog to Kabe's,

$$\min \sum_{i,j=1}^n \frac{(K_{ij} - K_{ij}^*)^2}{K_{ii}^* K_{jj}^*} = \min \|D^{-1}(K - K_s)D^{-1}\|_F^2$$

subject to  $KS = M_s \Omega^2$ ,  $K = K^t$ ,

and  $\text{sparse}(K) = \text{sparse}(K_s)$ ,

(3.7)

where  $D = \text{diag}(d_i) = \text{diag}(\sqrt{K_{ii}^*})$ .

These sparsity-preserving optimal-update techniques require the solution of an auxiliary problem to produce the update. Lagrange multipliers incorporate the constraints into an extended objective function. Minimization of the Lagrangian function produces a system of linear equations to solve for the Lagrange multipliers and an update equation for the stiffness matrix. The auxiliary problem has the dimension of the number of measured modes,  $p$ , times the number of original model degrees-of-freedom,  $n$ , or  $np \times np$ . While the auxiliary problem in the stiffness matrix adjustment method of Kabe [R-85] is symmetric and indefinite, the auxiliary problem of the second method is symmetric and positive semi-definite, permitting solution without a large storage requirement and with less computational effort.



With the solution of the auxiliary problem, elements of the adjusted stiffness matrix are formed from the original stiffness matrix as

$$K_{ij} = K_{ij}^a + d_i d_j [(P_i DS \Gamma)_j + (P_j DS \Gamma)_i] \text{ for } i, j = 1, 2, \dots, n. \quad (3.8)$$

The diagonal matrix  $P_i$  contains only ones and zeros to mask the mode shape vectors in  $S$  with the sparsity pattern of the  $i$ th row of  $K_a$ . The sparsity pattern of  $K_a$  does not consider zeros produced by structural symmetry (force cancellation), but rather represents physical connections in the structure between degrees of freedom of the model.

The partitioned vector  $\{\Gamma\}$  is the solution of the auxiliary system of equations, constructed from the original model and measured modal data as

$$F^T(I + \Pi)F \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \dots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \dots \\ \Delta_n \end{bmatrix}. \quad (3.9)$$

$F$  is a block diagonal matrix of weighted masked vectors using the same projection matrix  $P_i$ ,

$$F = \text{diag} [P_1 DS, P_2 DS, \dots, P_n DS] \quad (3.10)$$

which is combined with the identity matrix  $I$  and a reordering matrix  $\Pi$  to form the auxiliary problem coefficient matrix. The right-hand-side vector  $\{\Delta\}$  is another partitioned vector formed with the given data

$$\Delta_i = d_i^{-1} \begin{bmatrix} Y_{i1} \\ Y_{i2} \\ \dots \\ Y_{in} \end{bmatrix}, \quad (3.11)$$

weighting each row of  $Y = MS\Omega^2 - K_a S$ .

Implementation of the method can be accomplished by assembling the full symmetric positive semidefinite system and solving for  $\{\Gamma\}$ . However the dimension of the system ( $np \times$

np) can exceed computer storage capabilities if a large structure is considered. Iterative methods for solving Equation (3.8) take advantage of the repetitive substructure patterns and never assemble the coefficient matrix explicitly. A conjugate gradient method with diagonal preconditioning was used for solution of the auxiliary problem and required no more storage than that for the original stiffness matrix [R-110].

Computational effort for the auxiliary problem is much reduced in this second sparsity-preserving method, but preservation of sparsity and inconsistent data have been shown to be incompatible requirements. Experience has indicated that inconsistent data may lead to an updated model which is not positive definite even if the original model matrix is positive definite. Often this is not a problem if the initial model matrix is sufficiently accurate, but even then, noisy data may produce poor algorithm performance.

An underlying condition on the measured data is necessary for the existence of these solutions. In the case of stiffness matrix updates, measured mode shapes are presumed to be orthogonal to the original model mass matrix (which is presumed to be accurate). Environmental noise and imperfect modeling make it inevitable that this will not be exactly satisfied. Modifications to the measured modes to recover mass orthogonality [R-54] is an option.

A second option is to introduce the measured constraints as a soft constraint in the objective function, rather than requiring that the data be matched exactly as in the preceding formulations. We explored this option in an error-compensating version of the sparsity preserving update. Formulated as

$$\begin{aligned} \min \quad & \|D^{-1}(K - K_s)D^{-1}\|_F^2 + \|(KS - M_s S \Omega^2)[\alpha]\|_F^2 \\ \text{subject to } & K = K^T \text{ and } \text{sparse}(K) = \text{sparse}(K_s), \end{aligned} \quad (3.12)$$

where  $[\alpha]$  is a diagonal weighting matrix with  $\alpha_i$  as a weighting value for the  $i$ th measured mode. Here a matrix is found which is close to the original stiffness matrix and which is nearly consistent with the measured data, subject to symmetry and sparsity constraints. The weighting

values,  $\alpha_i$  are intended to balance the information from the initial model with that from the measured data by providing a confidence value for each measured mode. Here too an auxiliary problem must be solved, although the structure of the linear system is symmetric and positive definite, allowing the use of efficient iterative algorithms. Selection of appropriate weighting values to express the mode shape confidence is a difficult task, though.

Kammer [R-90] presented a sparsity-preserving stiffness matrix update that approaches the problem with the philosophy of the previously presented second viewpoint. In his method, sparsity is preserved by creating a vector of unknown stiffness elements from the unknown, nonzero elements in the stiffness matrix. In his method, the norm of the error vector generated using the measured data and the original model is minimized, with subsequent addition of a constraint to minimize the difference between the original and corrected stiffness matrices.

#### 4.0 New Developments in Optimal Matrix Adjustment

In this chapter, the second viewpoint is adopted for the development of new techniques for property matrix adjustment. Further details of the developments can be found in References R-191 and R-192, both supported by this grant.

These techniques are presented in a general format, using  $A$  to represent the property matrix. However, they can be applied for specific property matrices, as with the case of stiffness matrix adjustment using modal data (frequencies and mode shapes) to form the measured-data constraints. These ideas follow the early ideas of Brock [R-133], but here we consider the situation where the measured data presents a set of constraints that number less than the model order ( $p < n$ ).

The second viewpoint for in the optimal-update approach starts with a set of symmetric matrices which solve the problem of Equation (3.2), repeated here for reference,

$$\min \|AS - Y\|_F$$

$$\text{subject to } A = A^t.$$

$S$  and  $Y$  are  $n \times p$  matrices that use the test data to define the  $p$  constraints on the property matrix  $A$ . The columns of  $S$  are independent.

A symmetric matrix  $A$  which solves the problem above satisfies the Sylvester equation

$$ASS^t + SS^tA = YS^t + SY^t, \quad (4.1)$$

and can be determined as

$$A = UBU^t, \quad (4.2)$$

where

$U$  is a unitary matrix of the left singular vectors of  $S$ , from the singular value decomposition  $S = U\Sigma V^t$ , and  
 $B$  is an  $n \times n$  symmetric matrix.

$B$  is defined by

$$b_{ij} = \frac{c_{ij}}{\sigma_i^2 + \sigma_j^2} \quad \text{for each } i,j \text{ with } \min(i,j) \leq p, \quad (4.3)$$

where  $\{\sigma_i\}_{i=1}^p$  are the ordered nonzero singular values of  $S$  and  $C_{ij}$  are the elements of

$$C = U^t(YS^t + SY^t)U. \quad (4.4)$$

Details of the proofs associated with this result are included in R-192.

An alternate expression of the solution follows from partitioning the matrix  $B$  as

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{12}^t & B_{22} \end{bmatrix}, \quad (4.5)$$

using  $B = U^t A U$  and a partitioned  $U$ ,  $U = [U_1 \mid U_2]$ , with  $U_1$  as the  $n \times p$  submatrix that spans the range of  $S$ . With respective partitioning of  $C$ ,

$$B_{11} \text{diag}(\sigma_j^2) + \text{diag}(\sigma_i^2) B_{11} = C_{11}, \quad (4.6)$$

or elementwise,

$$(B_{11})_{ij} \sigma_j^2 + \sigma_i^2 (B_{11})_{ij} = (C_{11})_{ij}.$$

In addition,

$$\text{diag}(\sigma_i^2) B_{12} = C_{12}. \quad (4.7)$$

Several observations stem from this development. Updated property matrices which solve the stated problem are nonunique, defined only by the matrices  $B_{11}$  and  $B_{12}$  which include information from the measured-data constraints. The matrix  $B_{22}$  is arbitrary at this point, but imposition of additional constraints, such as closeness to an original model, fix  $B_{22}$  as well as  $A$ . Note that the conditions for positive definiteness of  $A$  are that  $B_{11}$  is positive definite and that  $B_{22}$  is chosen so that  $B_{22} = D + B_{12}^t B_{11}^{-1} B_{12}$  for an arbitrary positive definite matrix  $D$ .

With this observation, understanding is provided for another very different approach to the identification problem. The desired property matrix  $A$  and the measured data, represented in  $S$  and  $Y$  should satisfy

$$AS = Y, \quad (4.8)$$

so a set of linear equations can be constructed for the  $n(n+1)/2$  unknowns in  $A$  (once symmetry is imposed). Listing these unknowns in a long vector,  $\hat{a}$ , the linear system can be represented as

$$X\hat{a} = b, \quad (4.9)$$

where

$X$  is an  $n(n+1)/2 \times n(n+1)/2$  matrix, and  
 $b$  is an  $n(n+1)/2$  column vector,

both assembled with appropriate entries to replicate Equation (4.8). Finding the values of  $\hat{a}$  which satisfy this equation becomes computationally extensive for large models and may require a least squares solution. Even if the solution is obtained, no information about additional characteristics of  $A$ , including positive definiteness and closeness to an original model, is available from this approach. If the solution of this system is obtained through the Moore-Penrose generalized inverse,  $\hat{a} = X^\dagger b$ , then the assembled  $A$  would be the same  $A$  that results from Equations (4.2), (4.6) and (4.7) with the particular choice of  $B_{22} = 0$ . This choice for the arbitrary submatrix  $B_{22}$  precludes the property matrix  $A$  from being positive definite or even positive semidefinite. For applications to structure property matrices, positive definiteness or semidefiniteness is an important characteristic.

In the optimal-update formulation, the addition of a priori information in the form of an initial model can be presented as an additional constraint

$$\begin{aligned} \min \|AS - Y\|_F \\ \text{subject to } A = A^t \text{ and } \|A - A_s\|_F \text{ minimal.} \end{aligned} \quad (4.10)$$

Now, the solution is unique and given by  $A = UBU^t$  for  $B$  as defined by Equations (4.6) and (4.7) for  $B_{11}$  and  $B_{12}$ , respectively, and with

$$B_{22} = \frac{1}{2}U_2^t(Z_s + Z_s^t)U_2, \quad (4.11)$$

where  $A_s = \frac{1}{2}(Z_s + Z_s^t)$ .

Finally, adding a condition for positive semidefiniteness results in a formulation for the optimal-update problem as

$$\begin{aligned} \min \|AS - Y\|_F \\ \text{subject to } A = A^t, \text{ } A \text{ positive semidefinite,} \\ \text{and } \|A - A_s\|_F \text{ minimal.} \end{aligned} \quad (4.12)$$

Here,  $A = UBU^t$  for  $B_{11}$  and  $B_{12}$  as defined by Equations (4.6) and (4.7), respectively, while

$$B_{22} = B_{12}^t B_{11}^{-1} B_{12} + \text{pos}\left(\frac{1}{2}U_2^t(Z_s + Z_s^t)U_2 - B_{12}^t B_{11}^{-1} B_{12}\right), \quad (4.13)$$

where  $\text{pos}(W)$  is defined as the positive part of the  $n \times n$  matrix  $W$ . If  $U_w \Lambda U_w^t$  is the spectral factorization of  $\frac{1}{2}(W + W^t)$  with  $\Lambda = \text{diag}(\lambda_k)$ , then

$$\begin{aligned} \text{pos}(W) &= U_w \text{diag}(\eta_k) U_w^t \\ \text{with } \eta_k &= \max(0, \lambda_k) \quad \text{for } k=1, \dots, n. \end{aligned} \quad (4.14)$$

Final observations which result from this second viewpoint on optimal updates for property matrices include the ability to estimate the sensitivity of the solution matrices  $A$  to perturbations in the measured data  $S$  and  $Y$ . The closer the columns of  $S$  are to being an orthogonal set, the more stable the solution is to perturbations in the data [R-192].

The requirement for a specific sparsity pattern was not included in these new developments, but for stiffness matrix adjustment Kammer [R-90] has adopted this viewpoint and incorporated a sparsity preserving technique.

## 5.0 Evaluation Studies

In this section we present a summary of the evaluation studies that have been conducted for secant-method and optimal-update identification. Further details of these studies are included in all of the references supported by the grant. In particular, References R-112, R-107 and R-109 are summarized below.

### 5.1 Orthogonal Procrustes Expansion

Mode shape expansion is necessary to preserve the structure of the finite element model of a truss to enable damage location to a specific truss element. The process used for the estimation of the unmeasured degrees of freedom is critical to the performance of the identification algorithm, particularly to optimal-update identification techniques which are sensitive to inconsistencies in the mode shape data.

Optimal mode shape expansion, model reduction and mode shape orthogonalization are related to the problem of optimal-update identification. Within the framework of optimal updates, new expansion and orthogonalization techniques were developed under this research grant as well. Reference R-112 presents the details of this development and the results of the evaluation studies of these techniques. Both simulated studies with a small spring-mass problem and studies with experimental data from a laboratory truss structure were performed.

Orthogonal-Procrustes expansion is a technique which simultaneously expands and orthogonalizes the mode shape vectors to produce the mass-orthogonal, full mode shapes needed for structure identification. This technique was compared to several previously published techniques with the following results:

1. Orthogonal Procrustes expansion produced estimates of the full mode shape vectors which were calculated efficiently, simultaneously and ultimately orthogonal with respect to the model mass matrix.



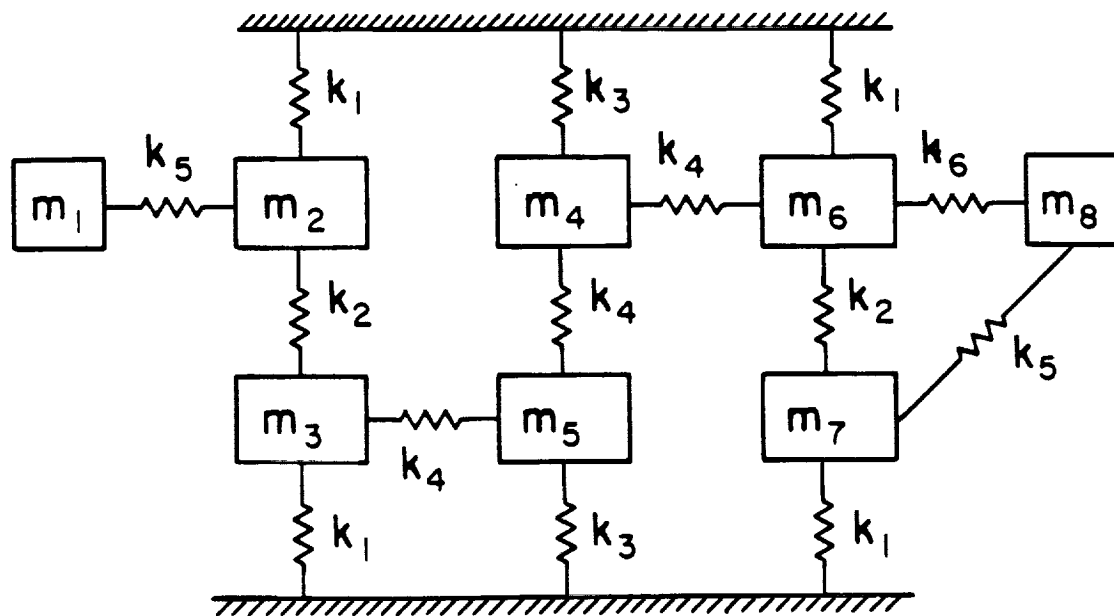
2. Dynamic expansion (as described by Berman and Nagy, R-66) with subsequent orthogonalization using the technique of Baruch and Bar Itzhack [R-54], produced results that were comparable to those using Orthogonal Procrustes expansion. Extensive computations are required to produce a large matrix inverse, though.
3. Sign errors in the small-displacement degrees of freedom that resulted from Orthogonal-Procrustes expansion in some cases were not as frequent with dynamic expansion and subsequent orthogonalization. These sign errors present a problem of inconsistent data to sparsity preserving optimal-update techniques.

## 5.2 Evaluation Studies with a Spring-Mass Problem

To illustrate characteristics of the identification algorithms and to provide common examples for comparison, a selection of identification algorithms are applied to two demonstration problems. The first problem is a widely-used spring-mass example [R-85]. It was used here for the purposes of investigating, on a small problem, ideas for mode shape expansion and techniques for structure identification. Alternate versions of Kabe's original problem were used, to examine updating for a local discrepancy, analogous to a damage detection situation.

Kabe's eight degree-of-freedom spring-mass problem is shown in Figure 5.1, which includes the stiffness and mass values for the exact model. This problem presents a challenging situation for structure identification in that stiffness values of various magnitudes are included. Closely-spaced frequencies are exhibited as well.

Variations of Kabe's original problem were used. Rather than the initial model he used, which had incorrect values for all of the connecting springs, initial models used for most of the studies were only incorrect for the spring between two masses, i.e. the spring between masses 3 and 5, for example. A value of 500, five times that of the exact spring, was assumed in this trial. The large difference in the initial value is more representative of a situation of damage detection, because often the initial system experiences a large, local reduction of stiffness when damaged.



$$\begin{aligned}
 k_1 &= 1000 & m_1 &= 0.001 \\
 k_2 &= 10 & m_8 &= 0.002 \\
 k_3 &= 900 & m_j &= 1.0 \quad j = 2, 7 \\
 k_4 &= 100 \\
 k_5 &= 1.5 \\
 k_6 &= 2.0
 \end{aligned}$$

Figure 5.1 Kabe's Spring-Mass Problem

Several different studies were performed with the various trial situations of Kabe's problem. Expansion techniques were evaluated and reported in reference R-112 as one. Mode selection studies for best identification performance was another. Results of these studies appear in several publications supported by this grant. A summary of some of the results follows:

1. Mode selection was investigated by using various combinations of available modes as data in the identification process. At first mode selection was accomplished by engineering judgement - selecting modes that exhibited the largest frequency difference with respect to those predicted by the initial model. Modes were also selected through the use of mode shape correlation, actually lack of it, with respect to the initial model modes. In many cases, these two sets - selected by frequency difference and selected by mode shape difference - are identical. But often they are not. In the small studies, use of mode shape correlation as a means of selection is indicated as a promising option in many cases. However, modes that contain extreme errors also may be poorly correlated. A combined approach which uses frequency differences and mode shape differences should be explored.
2. Related to the idea of mode selection is the problem of sensor placement, or which degrees of freedom to measure to have the best data for identification. In the small studies, arbitrary selection of measured degrees of freedom led to the conclusion that this issue is one of major importance for identification performance. Optimal-update techniques are sensitive to mode shape information and poor mode shape data as a result of unwise measurement locations seems common. Research and more practical experience is needed in this area.
3. Initial model correlation was studied as well. For these small identification problems there was no indication that the initial model of the stiffness matrix is critical, with the exception of the sparsity pattern. Large and varied initial model errors are identified with the application of fairly accurate, full modes. However, use of the initial model to esti-

mate the full mode shape vector does lead to poor performance in some cases. Sensor placement is a critical factor in this result as well.

4. Finally, data accuracy was examined in several studies. Generally, the identification algorithms tolerate up to 10 percent random noise added to the mode shape vectors. For sparse updates, one symptom of poor data is a non-positive-definite result.

### 5.3 Evaluation Studies with a Laboratory Truss

A focus of the research effort was to apply these techniques to the problem of identification and damage detection of a laboratory truss structure. Experiments and preliminary results are described in detail in references R-107 and R-109. A final summary of these results is in preparation to be a journal publication.

## 6.0 Summary

Efforts for development and evaluation of secant methods for structure identification have produced understanding in a broad area we termed optimal-update identification or optimal matrix adjustment. This is the major result of this grant effort. Constrained optimization formulations for updating the property matrices of a model of a structure using measured dynamic response data have been developed and examined. Another important result is that these techniques are also applicable to other problems in which multiple constraints can be specified for a matrix updating problem, i.e. control gain calculations, for example.

Evaluation of these techniques for the problem of damage detection is still in progress, however. Improved understanding resulted from the development efforts and evaluation studies under this grant but additional efforts were defined as a result. Inconclusive results for the damage location problem for the laboratory truss structure, in comparison with the ability to locate damage in simulated problems, has shown the need for further understanding of, and improved techniques for handling, inconsistent modal data. In particular, attention to the problem of sensor placement, among others, is needed to produce the best mode shape information for the identification process.

With the understanding and experience developed in this research effort, further advances for the application of damage location are possible. At this time, the most important addition to the knowledge base is experience with various real-data situations. This, in concert with understanding of other approaches for structure identification, will enable eventual implementation of identification algorithms for damage location.

During the course of this research, an undergraduate researcher was employed to help with the evaluation studies. Cameron A. Bryant will graduate with a BSCE, May 1991.

As a result of this research effort, ten publications have, or will, appear. In the following section, these publications are listed by type in reverse chronological order. Abstracts are reproduced where they are available.

Journal Articles

1. Smith, S.W. and P.E. McGowan, "Locating Damage in a Truss Using Modal Tests," to be submitted for consideration to the AIAA Journal

On-orbit assessment of large flexible space truss structures can be accomplished, in principle, with dynamic response information, structural identification and model correlation techniques which produce an adjusted mathematical model. In this approach for damage location, an optimal update of the structure model is formed using the response data, then examined to locate damage members. An experiment designed to demonstrate the performance of the on-orbit assessment approach uses a laboratory truss structure which exhibits characteristics expected for large space truss structures. Vibration experiments were performed to generate response data for the damaged truss. This paper describes the damage location approach, analytical work performed in support of the vibration tests, the measured response of the test article and damage location results.

2. Beattie, C.A. and S.W. Smith, "Optimal Matrix Approximants in Structural Identification," Journal of Optimization Theory and Applications, to appear.

Problems of model correlation and system identification are central in the design, analysis and control of large space structures. Of the numerous methods that have been proposed, many are based on finding minimal adjustments to a model matrix sufficient to introduce some desirable quality into that matrix. In this work, several of these methods are reviewed, placed in a modern framework, and linked to other previously known ideas in computational linear algebra and optimization. This new framework provides a point of departure for a number of new methods which are introduced here. Significant among these is a method for stiffness matrix adjustment which preserves the sparsity pattern of an original matrix, requires comparatively modest computational resources, and allows robust handling of noisy modal data. Numerical examples are included to illustrate methods presented herein.

3. Smith, S.W. and C.A. Beattie, "Secant-Method Adjustment for Structural Models," AIAA Journal, Vol. 29, No. 1, January 1991, pp. 119-126.

On-orbit testing will be required for final tuning and validation of any mathematical model of large space structures. Identification methods using limited response data to produce optimally adjusted property matrices seem ideal for this purpose, but difficulties exist in that application of previously published methods to large space truss structures. This article presents new stiffness matrix adjustment methods that generalize optimal-update secant methods found in quasi-Newton approaches for nonlinear optimization. Many aspects of previously published methods of stiffness matrix adjustment may be better understood within this new framework of secant methods. One of the new methods preserves realistic structural connectivity with minimal storage requirements and computational effort. A method for systematic compensation for errors in measured data is introduced that also preserves structural connectivity. Two demonstrations are presented to compare the new methods' results to those of previously published techniques.

Conference Proceedings (\* denotes presenter)

1. Smith, S.W.\* and C.A. Beattie, "Optimal Identification Using Inconsistent Modal Data," to be presented at the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics and Materials Conference, Baltimore, Maryland, April 8-10, 1991.

Structure identification is the process of using measured response to produce a mathematical model that accurately represents the structure's dynamic characteristics. This capability is essential for large space structures which can not be fully assembled and ground tested before their assembly on-orbit. Consequently, various approaches have been adopted to address this need and within each of these general approaches, numerous techniques have been developed. This work examines techniques under the general approach of optimal-update identification which produce optimally adjusted, or updated, property matrices (i.e., mass, stiffness and damping matrices) to more closely match the structure modal response. For practical applications, the techniques must perform when the modal response is inconsistent with other constraints on the desired model. Here we present an alternate view of the optimal-update problem that leads to new techniques for addressing inconsistent data. Viewpoints used for previously published techniques are also examined to explore in optimal-update identification.

2. Beattie, C.A.\* and S.W. Smith, "Matrix Approximation in the Identification of Vibrating Structures," Proceedings of the International Conference on New Developments in Structural Mechanics, Catania, Italy, July 2-5, 1990.

Numerous methods have been proposed for the adjustment of analytical models of vibrating structures in order to incorporate experimentally obtained information. Often this involves a modification to a model property matrix that is minimal in some appropriate sense, yet sufficient to induce consistency with experimental data. A naive formulation leads to the solution of a large unstructured linear system requiring (potentially) an enormous amount of computation and yielding no insight into other qualities one may wish to preserve in the material property matrix. We provide a path of development here that maintains the matrix theoretic aspects of the approximation problem and permits a substantial reduction in computational complexity while providing insight into circumstances that yield positive definite approximants. We show how these techniques can be applied to identify structural damping matrices and provide a simple illustrative example.

3. Smith, S.W.\* and C.A. Beattie, "Simultaneous Expansion and Orthogonalization of Measured Modes for Structure Identification," Proceedings of the AIAA SDM Dynamics Specialist Conference, Long Beach, California, April 5-6, 1990, pp. 261-270.

Tests of large structures on-orbit will be performed with measurements at a relatively few structure points. Values for the unmeasured degrees of freedom (dofs) can be estimated based on measured dofs and analytical model dynamic information. These "expanded" mode shapes are useful for optimal-update identification and damage location as well as test/analysis correlation. A new method of expansion for test mode shape vectors is developed from the orthogonal Procrustes problem from computational linear algebra. A subspace defined by the set of measured dofs is compared to a subspace defined by modes shapes from an analytical model of the structure. The method simultaneously expands and orthogonalizes the mode shape vectors. Two demonstration problems are used to compare the new method to current expansion techniques. One demonstration uses test data from a laboratory scale-model truss structure. Performance of the



new method is comparable or superior to that of the previous expansion methods which require separate orthogonalization.

4. McGowan, P.E.\*, S.W. Smith, and M. Javeed, "Experiments for Locating Damaged Members in a Truss Structure," Proceedings of the 2nd USAF/NASA Workshop on System Identification and Health Monitoring of Precision Space Structures, Pasadena, California, March 27-29, 1990.

This paper summarizes experiments performed to demonstrate and evaluate a previously developed method for locating damage in truss structures. A review of the damage location method and its application to large space truss structures is presented. Two focus test articles, which have been developed for the Dynamic Scale Model Technology (DSMT) project at NASA Langley, are described. Analytical studies show the sensitivity of a complex, dynamically scaled model of space station to damaged members. Vibration experiments were performed on a scale model truss section with and without damaged members to generate modal data required for the damage detection algorithm. Experimental results from different cases are included to validate the damage location method.

5. Smith, S.W.\* and P.E. McGowan, "Locating Damaged Members in a Truss Structure Using Modal Test Data: A Demonstration Experiment," NASA-TM-101595, April 1989. (also presented at the 30th Structures, Structural Dynamics and Materials Conference, Mobile Alabama, April 1989)

On-orbit assessment of large flexible space truss structures can be accomplished, in principle, with dynamic response information, structural identification and model correlation techniques which produce an adjusted mathematical model. In a previously developed approach for damage location, an optimal update of the structure model is formed using the response data, then examined to locate damage members. An experiment designed to demonstrate and verify the performance of the on-orbit assessment approach uses a laboratory scale model truss structure which exhibits characteristics expected for large space truss structures. Vibration experiments were performed to generate response data for the damaged truss. This paper describes the damage location approach, analytical work performed in support of the vibration tests, the measured response of the test article and some preliminary results.

6. Smith, S.W.\* and C.A. Beattie, "Secant-Method Matrix Adjustment for Structural Models," Proceedings of the AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics, and Materials (SDM) Conference, Mobile, AL, April 1989, pp. 1041-1051.

On-orbit testing will be required for final tuning and validation of any mathematical model of large space structures. Identification methods using limited response data to produce an optimally adjusted stiffness matrix seem ideal for this purpose, but difficulties arise in their application to large space truss structures. Typically, either unrealistic physical connectivity is produced in the adjusted model or massive storage and computational effort is required to preserve realistic connectivity. This paper presents new methods of stiffness matrix adjustment generalizing optimal-update secant methods from quasi-Newton approaches for nonlinear optimization. One of the new methods, the MSMT method, preserves realistic structural connectivity with minimal storage and computational effort, while an extension of the MSMT method allows compensation for errors in

measured data. Within the new framework of secant updates, aspects of previously used methods of stiffness matrix adjustment are better understood as well. Two demonstrations are presented to compare the new methods' results to those of previously published techniques.

#### Monograph Chapter

1. Zimmerman, D.C. and S.W. Smith, "Model Refinement and Damage Location for Intelligent Structures," to appear in Intelligent Structural Systems, Ed. H. S. Tzou., 1991.

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